1

1.a

$$y = 10 + 3x_1 - 1.5x_1^2 + 5x_2 - 2x_2^2 + 2x_1x_2$$
$$y_1 = 3 - 3x_1 + 2x_2$$
$$y_{12} = 2$$

Since $y_{12} = 2 > 0$, x_1 and x_2 are complementary

1.b

$$y_1 = 3 - 3x_1 + 2x_2 \quad (1)$$
$$y_1(tx_1, x_2) = 3 - 3tx_1 + 2x_2 \quad (2)$$
$$y_1(tx_1, x_2) = 3 + 2x_2 - 3tx_1 \quad (3)$$

$$y_1 = 3 - 3x_1 + 2x_2 \quad (1)$$
$$ty_1(x_1, x_2) = t(3 - 3x_1 + 2x_2) \quad (2)$$
$$ty_1(x_1, x_2) = t(3 + 2x_2) - 3tx_1 \quad (3)$$

Therefore, since $y_1(tx_1, x_2) < ty_1(x_1, x_2)$, the production function $y = 10 + 3x_1 - 1.5x_1^2 + 5x_2 - 2x_2^2 + 2x_1x_2$ has decreasing returns to scale and the government should decrease the funding for public research. However, we can find the optimal returns of investment through the first order condition as such:

$$y_1 = 3 - 3x_1 + 2x_2 = 0 \quad (1)$$
$$x_1 = \frac{3 + 2x_2}{3} \quad (2)$$

Therefore, optimal returns of investment for marginal productivity depends on the amount of investment in x_2. The government may choose to fund public research up until the point of $x_1 = \frac{3 + 2x_2}{3}$ for the optimal amount of marginal productivity. This means that the amount of government investment into public research would depend on the amount of research investment from the industry in x_2.

2

2.a

$$y(t_{x1}, t_{x2}) = -1.5(tx_1)^2 - 2(tx_2)^2 + 2(tx_1)(tx_2) \quad (1)$$
$$= t^2(-1.5x_1^2 + 2x_2 + 2x_1x_2) \quad (2)$$

Since $k = 2 > 1$, this function has increasing returns

2.b

$$y(t_{x1}, t_{x2}) = 5(tx_1)^{0.2}(tx_2)^{0.2} \quad (1)$$
$$= t^{0.4}x_1^{0.2}x_2^{0.2} \quad (2)$$

Since $k = 0.2 < 1$, this function has decreasing returns

2.c

$$y(t_{x1}, t_{x2}) = 5(tx_1)^{0.2}(tx_2)1.2 \quad (1)$$
$$= t^{1.4}5x_1^{0.2}x_2^{1.2} \quad (2)$$

Since $k = 1.4 > 1$, this function has increasing returns

2.d

$$y = 5(tx_1)^{0.2}(tx_2)^{0.8} \quad (1)$$
$$= t5x_1^{0.2}x_2^{0.8} \quad (2)$$

Since $k = 1$, this function has constant returns

3

3.a

$$y = 5x_1^{0.3}x_2^{0.6} \quad (1)$$
$$\ln y = \ln(5x_1^{0.3}x_2^{0.6}) \quad (2)$$
$$= \ln 5 + \ln x_1^{-.3} + \ln x_2^{0.6} \quad (3)$$
$$= \ln 5 + 0.3 \ln x_1 + 0.6 \ln x_2 \quad (4)$$

Therefore, $E_{y,x_1} = 0.3$ and $E_{y,x_2} = 0.6$

3.b

$$RTS = \frac{y_1}{y_2} \quad (1)$$
$$= \frac{5(0.3)x_1^{-0.7}x_2^{0.6}}{5(0.6)x_1^{0.3}x_2^{-0.4}} \quad (2)$$
$$= \frac{1}{2}\frac{x_2}{x_1} \quad (3)$$

Therefore, the RTS of capital for labor is 0.5, or two units of capital for one unit of labor

3.c

As we found in part 3(c), since the RTS of capital for labor is 0.5, if we reduce capital by one unit from 10 to 9, we must increase labor by two units from 10 to 12 in order to maintain the same level of output. Of course, we can check our work if we plug in 10 units of capital and 10 units of labor into our original production function, decrease capital to 9, and then find the labor necessary for the original production function and the decreased capital to be the same:

$$y(x_1, x_2) = 5x_1^{0.3}x_2^{0.6} \quad (1)$$
$$y(10, 10) = 5(10)^{0.3}(10)^{0.6} \quad (2)$$
$$y(10, 10) = 5(10^{0.9}) \quad (3)$$

$$y(9, x_2) = 5(9)^{0.3}x_2^{0.6} \quad (1)$$

Setting $y(10, 10)$ and $y(9, x_2)$ equal to each other and solving for x_2, we get:

$$5(10^{0.9}) = 5(9)^{0.3}x_2^{0.6} \quad (1)$$
$$10^{0.9} = 9^{0.3}x_2^{0.6} \quad (2)$$
$$10^{9/10} = 9^{3/10}x_2^{3/5} \quad (3)$$
$$x_2 = (\frac{10^{9/10}}{9^{3/10}})^{5/3} \quad (4)$$
$$= \frac{10^{3/2}}{9^{1/2}} \quad (5)$$
$$x_2 = 10.5 \quad (6)$$

Therefore, labor should increase by $\frac{1}{2}$ if capital decreases by 1.

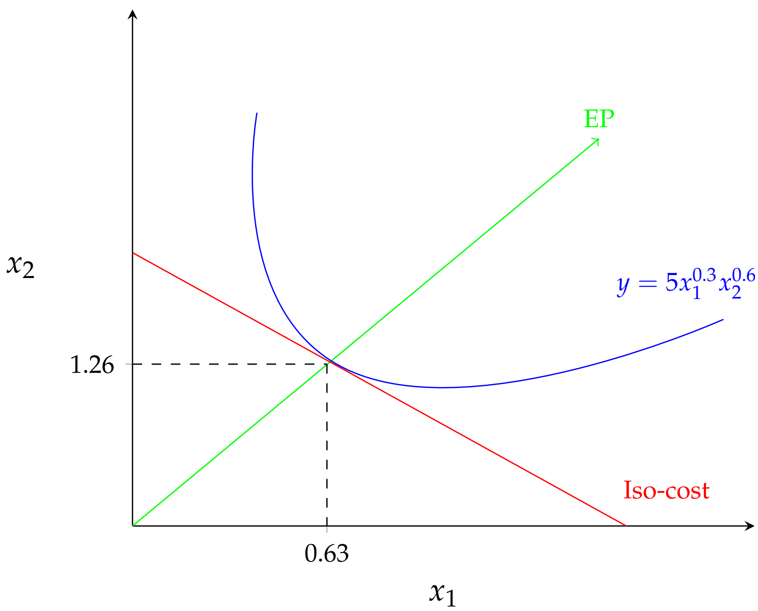
3.d

Given that $y = 5$, we can find the optimal values of x_1 and x_2 given this output of 5:

$$5 = 5x_1^{0.3}x_2^{0.6} \quad (1)$$
$$2x_1 = x_2 \quad (2)$$

Substituting this back into our production, we can get our least cost capital and labor quantities:

$$5 = 5x_1^{0.3}x_2^{0.6} \quad (1)$$
$$1 = x_1^{0.3}(2x_1)^{0.6} \quad (2)$$
$$1 = 2^{0.6}x_1^{0.9} \quad (3)$$
$$1 = 2^{3/5}x_1^{9/10} \quad (4)$$
$$x_1 = (\frac{1}{2^{3/5}})^{10/9} \quad (5)$$
$$= 2^{-2/3} \quad (6)$$
$$x_1^* = 0.63 \quad (7)$$
$$x_2^* = 2 \times 0.63 = 1.26 \quad (8)$$
$$(9)$$



4

4.a

Given:

$$K = 10y(\frac{w}{r})^{0.3} \quad (1)$$
$$L = 5y(\frac{r}{w})^{0.7} \quad (2)$$

We can find the wage and rental rate elasticity of capital and labor by taking the natural log as such:

$$\ln K = \ln(10y(\frac{w}{r})^{0.3}) \quad (1)$$
$$= \ln 10 + \ln y + 0.3 \ln w - 0.3 \ln r \quad (2)$$

$$\ln L = \ln(5y(\frac{r}{w})^{0.7}) \quad (1)$$
$$= \ln 5 + \ln y + 0.7 \ln r - 0.7 \ln w \quad (2)$$

Since we now know that $E_{K,w} = 0.3 = \frac{\%\Delta K}{\%\Delta w}$ and $E_{L,w} = -0.7 = \frac{\%\Delta L}{\%\Delta w}$, we can find the effect of the percentage change of wage on capital and labor:

$$E_{K,w} = 0.3 = \frac{\%\Delta K}{\%\Delta w} \quad (1)$$
$$0.3 = \frac{\%\Delta K}{-20\%} \quad (2)$$
$$\%\Delta K = -6\% \quad (3)$$

$$E_{L,w} = 0.7 = \frac{\%\Delta L}{\%\Delta w} \quad (1)$$
$$-0.7 = \frac{\%\Delta L}{-20\%} \quad (2)$$
$$\%\Delta L = 14\% \quad (3)$$

4.b

$$E_{K,r} = -0.3 = \frac{\%\Delta K}{\%\Delta r} \quad (1)$$
$$-0.3 = \frac{\%\Delta K}{-10\%} \quad (2)$$
$$\%\Delta K = 3\% \quad (3)$$

$$E_{L,r} = 0.7 = \frac{\%\Delta L}{\%\Delta r} \quad (1)$$
$$0.7 = \frac{\%\Delta L}{-10\%} \quad (2)$$
$$\%\Delta L = -7\% \quad (3)$$

4.c

Since the wage and rental rate has risen the same percentage, the ratio between wage and rental rate remains the same, and we would expect the firm to stay with the same level of capital and labor inputs. If we plug in 30% into our equations from 4(a) and 4(b) we would get $21\% - 21\%$ for capital and $9\% - 9\%$ for labor, which results in no change for either capital and labor inputs.

5

5.a

$$\ln \frac{C}{p} = 3.6 + 0.8 \ln y + 0.5 \ln \frac{w}{p} \quad (1)$$
$$\ln C - \ln p = 3.6 + 0.8 \ln y + 0.4 \ln w - 0.4 \ln p \quad (2)$$
$$\ln c = 3.6 + 0.8 \ln y + 0.4 \ln w + 0.6 \ln p \quad (3)$$
$$e^{\ln c} = e^{3.6+0.8 \ln y+0.4 \ln w+0.6 \ln p} \quad (4)$$
$$C = e^{3.6} e^{0.8 \ln y} e^{0.4 \ln w} e^{0.6 \ln p} \quad (5)$$
$$C = e^{3.6} y^{0.8} w^{0.4} p^{0.6} \quad (6)$$

The generalized total cost function for a Cobb-Douglas is given as:

$$C(v, w, y) = By^{1/\alpha+\beta} v^{\alpha/\alpha+\beta} w^{\alpha/\alpha+\beta}$$

Since we know the returns to scale on a Cobb-Douglas is determined if $\alpha + \beta$ is greater to, equal to, or less than one, we can use our previously found cost function to find the returns to scale:

$$\frac{1}{\alpha + \beta} = 0.8 \quad (1)$$

$$\alpha + \beta = \frac{1}{0.8} \quad (2)$$

$$\alpha + \beta = 1.25 > 1 \quad (3)$$

Since $\alpha + \beta = 1.25 > 1$, this function has increasing returns

5.b

For labor

$$D_L = \frac{\partial C}{\partial w} \quad (1)$$

$$= e^{3.6}(0.4)y^{0.8}w^{-0.6}p^{0.6} \quad (2)$$

$$= e^{3.6}(0.4)(\frac{p}{w})^{0.6}y^{0.8} \quad (3)$$

For fuel

$$D_F = \frac{\partial C}{\partial p} \quad (1)$$

$$= e^{3.6}(0.6)y^{0.8}w^{0.4}p^{-0.4} \quad (2)$$

$$= e^{3.6}(0.6)(\frac{w}{p})^{0.4}y^{0.8} \quad (3)$$

5.c

Since now we know $D_F = e^{3.6}(0.6)(\frac{w}{p})^{0.4}y^{0.8}$, we can find the elasticity of fuel with respect to fuel price:

$$\ln D_F = \ln e^{3.6}(0.6)(\frac{w}{p})^{0.4}y^{0.8} \quad (1)$$

$$\ln D_F = \ln e^{3.6} + \ln 0.6 + 0.4 \ln w - 0.4 \ln p + 0.8 \ln y \quad (2)$$

Therefore,

$$E_{F,p} = -0.4 = \frac{\%\Delta F}{\%\Delta p} \quad (1)$$

$$-0.4 = \frac{\%\Delta F}{-10\%} \quad (2)$$

$$\%\Delta F = 4\% \quad (3)$$

We see that a 10% decrease in fuel price results in a 4% increase in the demand for fuel.